# GCE AS/A LEVEL

2305U30-1

Z22-2305U30-1

**THURSDAY, 26 MAY 2022 – AFTERNOON**

## FURTHER MATHEMATICS – AS unit 3
## FURTHER MECHANICS A

1 hour 30 minutes

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### ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

### INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take $g$ as 9·8ms$^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

### INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**Reminder**: Sufficient working must be shown to demonstrate the **mathematical** method employed.

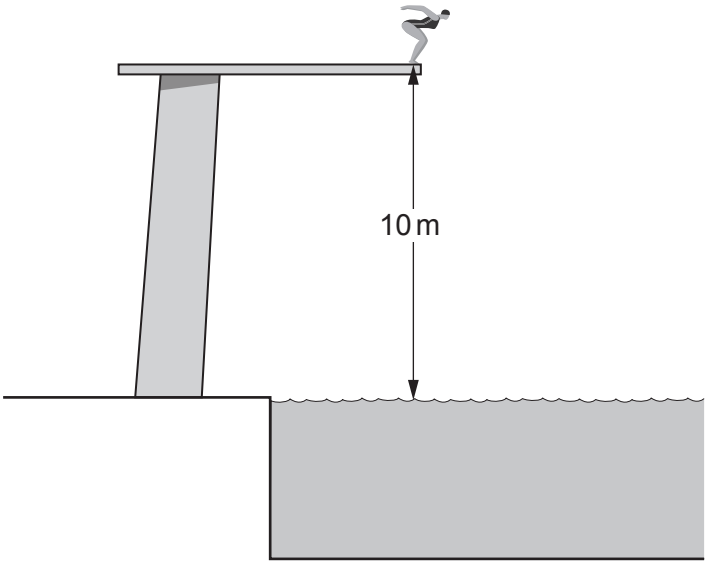
**1.** A particle of mass 1·2 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is fixed to a point *O* on a smooth horizontal surface. With the string taut, the particle moves on the surface with constant speed 8 ms$^{-1}$ in a horizontal circle with centre *O*.

(a) Find the angular velocity of the particle about *O*. [2]

(b) Calculate the tension in the string. [2]

**2.** The diagram below shows a woman standing at the end of a diving platform. She is about to dive into the water below.

The woman has mass 60 kg and she may be modelled as a particle positioned at the end of the platform which is 10 m above the water.




When the woman dives, she projects herself from the platform with a speed of 7·8 ms$^{-1}$.

(a) Find the kinetic energy of the woman when she leaves the platform. [2]

(b) Initially, the situation is modelled ignoring air resistance. By using conservation of energy, show that the model predicts that the woman enters the water with an approximate speed of 16 ms$^{-1}$. [6]

(c) Suppose that this model is refined to include air resistance so that the speed with which the woman enters the water is now predicted to be 13 ms$^{-1}$.
Determine the amount of energy lost to air resistance according to the refined model. [3]

**3.** Two spheres *A* and *B*, of equal radii, are moving towards each other on a smooth horizontal surface and collide directly. Sphere *A* has mass $4m$ kg and sphere *B* has mass $3m$ kg. Just before the collision, *A* has speed 9 ms$^{-1}$ and *B* has speed 3·5 ms$^{-1}$. Immediately after the collision, *A* has speed 1·5 ms$^{-1}$ in the direction of its original motion.

(a) Show that the speed of *B* immediately after the collision is 6·5 ms$^{-1}$. [3]

(b) Calculate the coefficient of restitution between *A* and *B*. [3]

(c) Given that the magnitude of the impulse exerted by *B* on *A* is 36 Ns, find the value of $m$. [3]

(d) Give a reason why it is not necessary to model the spheres as particles in this question. [1]

**4.** A particle *P* of mass 0·5 kg is in equilibrium under the action of three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

$$\mathbf{F}_1 = (9\mathbf{i} + 6\mathbf{j} - 12\mathbf{k})\text{ N} \quad \text{and} \quad \mathbf{F}_2 = (6\mathbf{i} - 7\mathbf{j} + 3\mathbf{k})\text{ N}.$$



(a) Find the force $\mathbf{F}_3$. [2]

(b) Forces $\mathbf{F}_2$ and $\mathbf{F}_3$ are removed so that *P* moves in a straight line *AB* under the action of the single force $\mathbf{F}_1$. The points *A* and *B* have position vectors $(2\mathbf{i} - 9\mathbf{j} + 7\mathbf{k})$ m and $(8\mathbf{i} - 5\mathbf{j} - \mathbf{k})$ m respectively. The particle *P* is initially at rest at *A*.

(i) Verify that $\mathbf{F}_1$ acts parallel to the vector **AB**.

(ii) Find the work done by the force $\mathbf{F}_1$ as the particle moves from *A* to *B*.

(iii) By using the work-energy principle, find the speed of *P* as it reaches *B*. [7]

# TURN OVER

**5.** One end of a light elastic string, of natural length 2·5 m and modulus of elasticity $30g$ N, is fixed to a point *O*. A particle *P*, of mass 2 kg, is attached to the other end of the string. Initially, *P* is held at rest at the point *O*. It is then released and allowed to fall under gravity.

(a) Show that, while the string is taut,

$$v^2 = g\left(5 + 2x - 6x^2\right),$$

where $v$ ms$^{-1}$ denotes the velocity of the particle when the extension in the string is $x$ m. [6]

(b) Calculate the maximum extension of the string. [3]

(c) (i) Find the extension of the string when *P* attains its maximum speed.

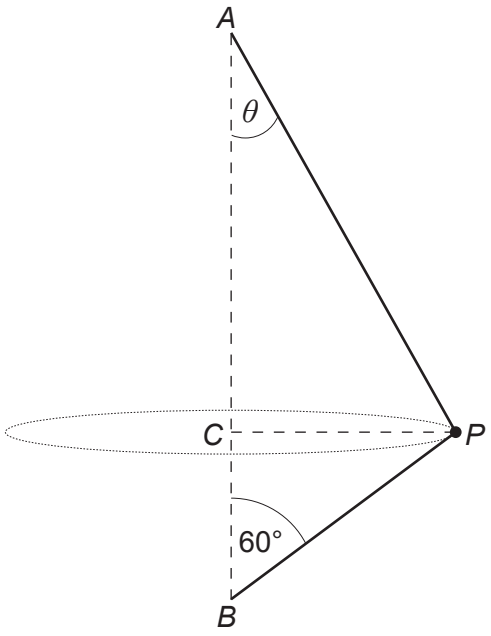
(ii) Hence determine the maximum speed of *P*. [5]

**6.** A vehicle of mass 3500 kg is moving up a slope inclined at an angle $\alpha$ to the horizontal. When the vehicle is travelling at a velocity of $v$ ms$^{-1}$, the resistance to motion can be modelled by a variable force of magnitude $40v$ N.

(a) Given that $\sin\alpha = \frac{3}{49}$, calculate the power developed by the engine at the instant when the speed of the vehicle is 25 ms$^{-1}$ and its deceleration is 0·2 ms$^{-2}$. [5]

(b) When the vehicle's engine is working at a constant rate of 40 kW, the maximum speed that can be maintained up the slope is 20 ms$^{-1}$. Find the value of $\alpha$. Give your answer in degrees, correct to one significant figure. [5]

**7.** The diagram below shows a particle *P*, of mass 2·5 kg, attached by means of two light inextensible strings fixed at points *A* and *B*. Point *A* is vertically above point *B*. *BP* makes an angle of 60° with the upward vertical and *AP* is inclined at an angle $\theta$ to the downward vertical where $\cos\theta = 0{\cdot}8$. The particle *P* describes a horizontal circle with constant angular speed $\omega$ radians per second about centre *C* with both strings taut.




The tension in the string *BP* is 39·2 N.

(a) Calculate the tension in the string *AP*. [4]

(b) Given that the length of the string *AP* is 1·5 m, find the value of $\omega$. [5]

(c) Calculate the kinetic energy of *P*. [3]

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